Scalable Clifford-Based Classical Initialization for the Quantum Approximate Optimization Algorithm

Dhanvi Bharadwaj, Yuewen Hou, Guang-Yi Li, Gokul Subramanian Ravi
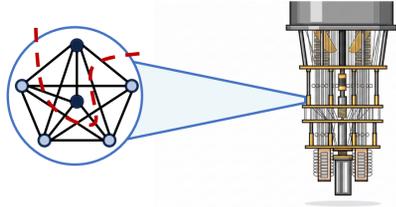
University of Michigan

arXiv!

Initialization in QAOA

Motivation : The Quantum Approximate Optimization Algorithm (QAOA) is a leading candidate to approximately solve combinatorial optimization problems, but its performance and cost depends heavily on the quality of initial variational parameters.

Challenges :

• Finding optimal initial parameters (γ, β) is expensive and suffers from the "Barren Plateau" phenomenon in high-dimensional landscapes.

• Existing initialization strategies are problem-specific, lack scalability for diverse problem types (QUBO, PUBO, PCBO), or are ineffective for weighted instances.

Clifford-Based Search

Core Idea: Utilize the Gottesman-Knill theorem, which allows Clifford circuits to be simulated efficiently on classical computers.
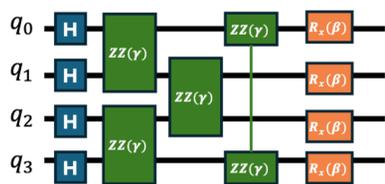
Directly applying successful Clifford-based initialization methods for VQE, to standard QAOA proves ineffective due to its limited ansatz expressivity.
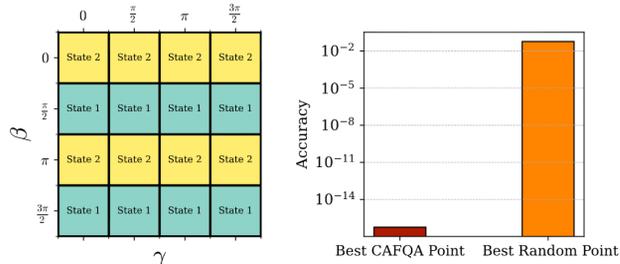
Hence, we enhance the expressivity of the Clifford search space by freeing the parameters of the QAOA Ansatz.
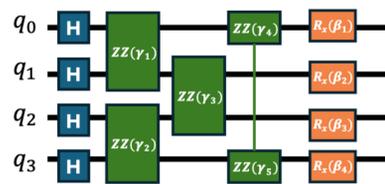
$p \rightarrow$ Number of QAOA repetitions
$m \rightarrow$ Number of clauses
$n \rightarrow$ Number of qubits

(a) Traditional QAOA Ansatz

$(4)^{2 \times p}$ **combinations**
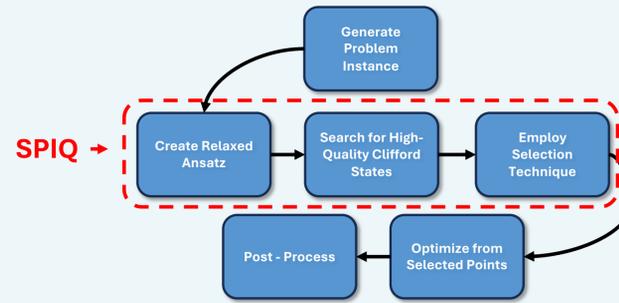
(b) MA-QAOA Ansatz

$(4)^{(m+n) \times p}$ **combinations**

Our Contributions

We develop **SPIQ**-a scalable, Clifford-driven initialization framework for QAOA that:

• **Expands search space** by leveraging the enhanced expressivity of Multi-Angle QAOA (ma-QAOA).

• **Identifies high-quality parameters** within the Clifford-simulable subspace using an efficient Genetic Algorithm.

• **Employs strategic selection methods**, to choose diverse, high-potential Clifford points for robust multi-start quantum optimization.

SPIQ →
Generate Problem Instance
Create Relaxed Ansatz
Search for High-Quality Clifford States
Employ Selection Technique
Optimize from Selected Points
Post - Process

Point Selection Strategies

To prevent the algorithm from stalling in stationary regions, we propose a multi-start strategy using two distinct point-selection methods to carefully seed the QAOA optimization.

Seeds trapped in flat or sub-optimal regions

Landscape-aware seeding for superior solutions
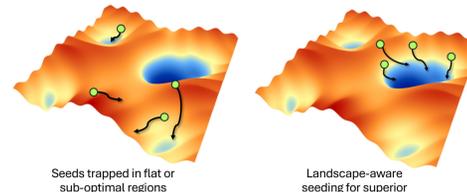
Fixed-Interval Selection : Candidate points are sorted by their expectation value, and seed points are selected at regular, evenly spaced intervals from this list.

K-GAPS (K-means Gradient-Aware Point Selection): A novel, landscape-aware strategy designed to guarantee both spatial diversity and high optimization potential. It operates in two steps:

• **Spatial Clustering (Embedded K-means):** To account for angular periodicity, Clifford angles are embedded onto a unit circle as $(\cos(\theta), \sin(\theta))$. We apply K-means clustering to these coordinates to ensure seeds are drawn from geometrically distinct regions.

• **Gradient Filtering:** We discard points with near-zero gradients so the optimizer doesn't start in a plateau. Because the seeds are Clifford states, these gradient norms are computed extremely fast using $\pm \frac{\pi}{2}$ parameter shifts that remain within the classically simulable Clifford group.

Key Idea : Ensure starting points are not only high-quality and structurally diverse, but also situated on steep optimization curves primed for rapid convergence.
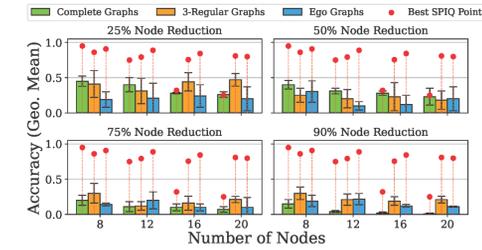
Performance Evaluation of SPIQ

Figure 1. Performance of SPIQ against RedQAOA on unweighted Max-Cut instances.
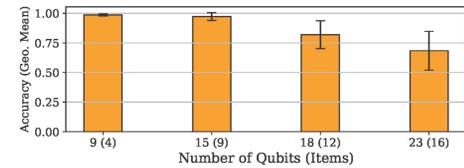
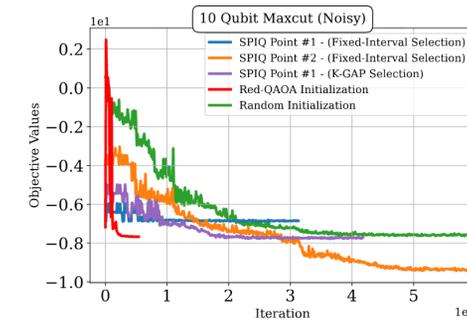**Figure 2.** SPIQ Accuracy evaluated on Knapsack (PCBO) instances.

Figure 3. Effect of initialization choice that leads to higher quality solution.
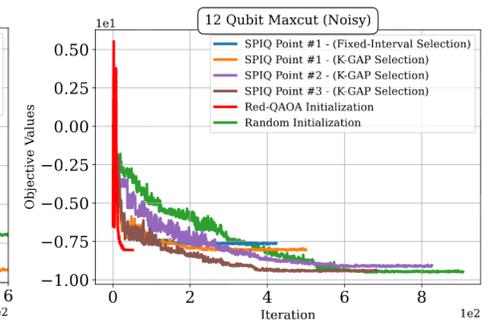
Figure 4. Effect of initialization choice that leads to faster convergence.

Key Takeaways :

• We **enhance ansatz expressivity with ma-QAOA**, enabling effective classical search within the Clifford group and overcoming key limitations of standard QAOA.

• We **leverage Clifford-based search to identify near-optimal initializations**, significantly reducing quantum optimization cost.

• We **develop multi-start strategies** to drive better solutions, accelerating convergence and improving robustness to noise and local minima.

Acknowledgements and References

[1] Farhi et al., A Quantum Approximate Optimization Algorithm, *arXiv:1411.4028* (2014).
[2] Herrman et al., Multi-angle QAOA, *arXiv:2109.11455* (2021).
[3] Wang et al., Red-QAOA: Efficient optimization via circuit reduction, *ASPLOS* (2024).